

2 men catch 2 fish in 2 minutes. At this rate, how many men could catch 500 fish in 500 minutes?

INDIRECT ← DIRECT

Men	Fish	Minutes
2	2	2
$x = ?$	500	500

There is **DIRECT PROPORTION** between fish and minutes because as 1 goes up, the other does too.

Fewer men need more time to catch fish. More men need less time to catch fish.

∴ there is **INDIRECT PROPORTION** between number of men and fish. minutes.

Fraction of quantities in table:

Men

$$\frac{x}{2} \cdot \frac{2}{x} = \frac{2}{500} \times \frac{500}{2}$$

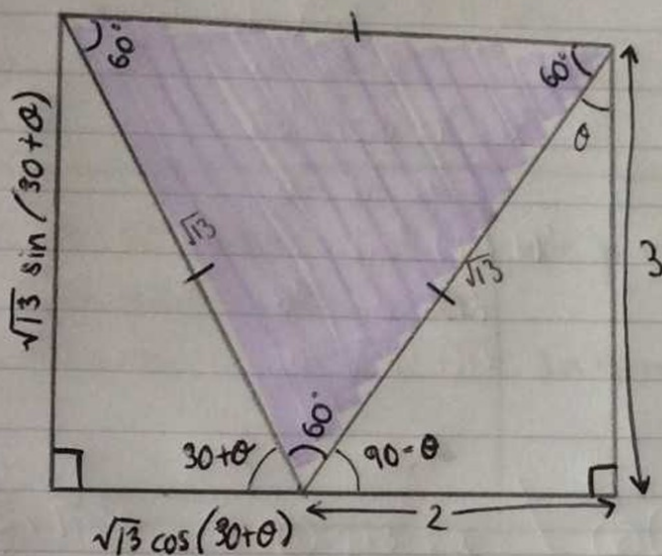
$$\frac{x}{2} \cdot \frac{2}{x} = 1 \rightarrow x \cdot 2$$

$$x = \underline{\underline{2 \text{ men}}}$$

If relation is **DIRECT**, keep number as it is. If it is **INDIRECT**, flip numbers in the fraction.

## Louise's Solution (SAK Tutor Base) - Diversity

which has the larger area? The pink equilateral or the combined right angle triangles?



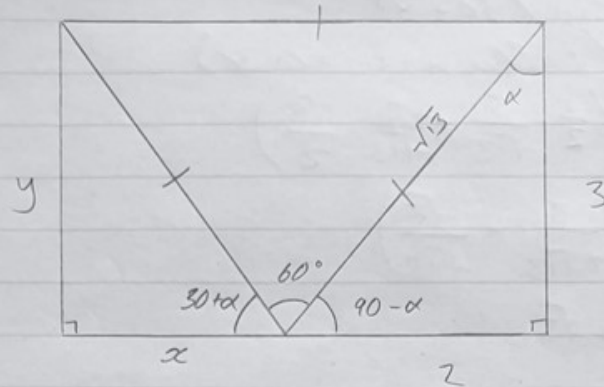
$$\theta = \tan^{-1} \left( \frac{2}{3} \right)$$

$$\begin{aligned} \sqrt{13} \cos(30+\theta) &= 1.598\ 076\ 211 \\ \sqrt{13} \sin(30+\theta) &= 3.232\ 050\ 808 \\ \text{area} &= \frac{2.582\ 531\ 754}{3} \\ &= 5.582\ 531 \end{aligned}$$

$$\text{equilateral} = 5.629\ 165\ 125$$

equilateral area > 2 right angle triangle area.

## Niall's Solution (CPC Tutor Base) - Opportunity



Equilateral Triangle side length

$$a^2 = 2^2 + 3^2$$

$$a = \sqrt{13}$$

$$\alpha = \arctan \frac{2}{3}$$

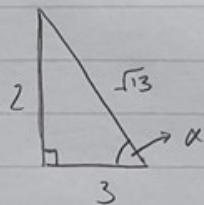
$$x = \sqrt{13} \cos(30 + \alpha)$$

$$y = \sqrt{13} \sin(30 + \alpha)$$

$$x = \sqrt{13} \cos(30 + \alpha)$$

$$= \sqrt{13} (\cos 30 \cos \alpha - \sin 30 \sin \alpha)$$

$$= \sqrt{13} \left( \frac{\sqrt{3}}{2} \cos(\tan^{-1}(\frac{2}{3})) - \frac{1}{2} \sin(\tan^{-1}(\frac{2}{3})) \right)$$



$$\cos \alpha = \frac{3}{\sqrt{13}}$$

$$\sin \alpha = \frac{2}{\sqrt{13}}$$

$$x = \sqrt{13} \left( \frac{\sqrt{3}}{2} \cdot \frac{3}{\sqrt{13}} - \frac{1}{2} \cdot \frac{2}{\sqrt{13}} \right)$$

$$= \sqrt{13} \left( \frac{3\sqrt{3}}{2\sqrt{13}} - \frac{1}{\sqrt{13}} \right)$$

$$= \frac{3\sqrt{3}}{2} - 1$$

## Niall's Solution (CPC Tutor Base) - Opportunity

$$\begin{aligned}y &= \sqrt{13} \sin(30 + \alpha) \\&= \sqrt{13} (\sin 30 \cos \alpha + \sin \alpha \cos 30) \\&= \sqrt{13} \left( \frac{1}{2} \cdot \frac{3}{\sqrt{13}} + \frac{2}{\sqrt{13}} \cdot \frac{\sqrt{3}}{2} \right) \\&= \sqrt{13} \left( \frac{3}{2\sqrt{13}} + \frac{\sqrt{3}}{\sqrt{13}} \right) \\&= \frac{3}{2} + \sqrt{3}\end{aligned}$$

Area of left triangle

$$\text{area} = \frac{xy}{2}$$

$$\begin{aligned}&= \left( \frac{3\sqrt{3}-2}{2} \right) \left( \frac{3+2\sqrt{3}}{2} \right) \cdot \frac{1}{2} \\&= \frac{9\sqrt{3} + 18 - 6 - 4\sqrt{3}}{8} \\&= \frac{12 + 5\sqrt{3}}{8}\end{aligned}$$

Area of right triangle

$$\text{area} = \frac{3 \cdot 2}{2}$$

$$= 3$$

Non-equilateral triangle sum

$$\begin{aligned}&= \frac{3 + \frac{12 + 5\sqrt{3}}{8}}{8} \\&= \frac{36 + 5\sqrt{3}}{8} \approx 5.582532\end{aligned}$$

Equilateral Triangle area

$$\text{area} = \frac{1}{2} \cdot \sqrt{13} \cdot \sqrt{13} \cdot \sin 60^\circ$$

$$= \frac{13}{2} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{13\sqrt{3}}{4} \approx 5.629165 \quad \therefore \text{The equilateral triangle is larger}$$